Calculus Warm Up #2-3

Use separation of variables to find the solution y = f(x) of the differential equation with initial condition f(0) = -2.

$$\frac{dy}{dx} = \frac{x+1}{y}$$

HW Questions: p. 579

45. $\displaystyle\sum_{n=1}^{\infty} \frac{n+10}{10n+1}$

47. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$$= \underset{n=1}{\left(1-\frac{1}{3}\right)} + \underset{n=2}{\left(\frac{1}{2}-\frac{1}{4}\right)} + \underset{n=3}{\left(\frac{1}{3}-\frac{1}{5}\right)} + \underset{n=4}{\left(\frac{1}{4}-\frac{1}{6}\right)} + \underset{n=5}{\left(\frac{1}{5}-\frac{1}{7}\right)}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

49. $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$

51. $\sum_{n=0}^{\infty} (1.075)^n$

53. $\sum_{n=2}^{\infty} \frac{n}{\ln n}$

55. $\sum_{n=1}^{\infty} \left(1 + \frac{k}{n}\right)^n$

$\lim_{n \to \infty} \left[\frac{n}{\ln n}\right] = \frac{\infty}{\infty}$

$\lim_{n \to \infty} \frac{1}{\frac{1}{n}} = \frac{1}{0} = \infty$

53. $\sum_{n=2}^{\infty} \frac{n}{\ln n}$

55. $\sum_{n=1}^{\infty} \left(1 + \frac{k}{n}\right)^n$

$A = P\left(1 + \frac{k}{n}\right)^{nt}$

$A = Pe^{kt}$

$\lim_{n \to \infty} \left(1 + \frac{k}{n}\right)^n = e^k \neq 0$

$\therefore$ Series diverges by the n^{th} term test.

$e = \lim_{x \to 0} (1 + x)^{1/x}$

$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

10.3 The Integral Test

p - Series

Harmonic Series

The Integral Test

If f is:
- positive for $x \geq 1$
- continuous
- decreasing

and: $f(n) = a_n$ (the n^{th} term)

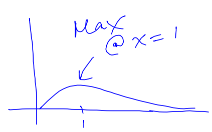
Then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$

Both Converge $\boxed{\text{or}}$ Both Diverge

Use the Integral Test to determine if the Series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1} \quad \text{so} \quad f(x) = \frac{x}{x^2+1}$$

check hypothesis for $x \geq 1$:



✓ • positive
✓ • continuous
✓ • decreasing

now integrate: $\frac{1}{2}\int_1^{\infty} \frac{2x}{x^2+1}\,dx$

$$\frac{1}{2}\lim_{b\to\infty}\int_1^{b} \frac{2x}{x^2+1}\,dx$$

$$\frac{1}{2}\lim_{b\to\infty}\Big[\ln(x^2+1)\Big]_1^{b}$$

$$\frac{1}{2}\lim_{b\to\infty}\left(\ln(b^2+1)-\ln 2\right)$$

$$\frac{1}{2}(\infty-\ln 2)$$

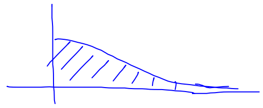
$$=\infty$$

∴ Series Diverges.

You try: Use the Integral Test to determine if the Series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1} \quad \text{so} \quad f(x) = \frac{1}{x^2+1}$$

check hypothesis for $x \geq 1$:



- ✓ positive
- ✓ continuous
- ✓ decreasing

then integrate: $\lim\limits_{b\to\infty} \int_1^b \frac{1}{x^2+1}\,dx$

$\lim\limits_{b\to\infty} \Big[\arctan x\Big]_1^b$

$\lim\limits_{b\to\infty} (\arctan b - \arctan 1)$

$\frac{\pi}{2} - \frac{\pi}{4}$

∴ Series Converges.

* But not to → $\boxed{\frac{\pi}{4}}$

The Integral Test does not give you the sum of the infinite series.

$\sum_{n=1}^{\infty} a_n$ Does not necessarily = $\int_1^{\infty} f(x)\,dx$

Ex: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \neq \int_1^{\infty} \frac{1}{x^2}\,dx = 1$

The sum of the series ≠ the value of the integral

p - Series

Definition: $\sum_{n=1}^{\infty} \frac{1}{n^p}$; p = a positive constant

A p - Series Converges if $p > 1$

Diverges if $0 < p \leq 1$

Our last example: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ Converges

Harmonic Series: A p - Series when p = 1

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

A p - Series Converges if $p > 1$

Diverges if $0 < p \leq 1$

Decide if the infinite series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$\lim_{b \to \infty} \int_2^b \frac{1}{x \ln x}\, dx$$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$\lim_{b \to \infty} \Big[\ln(\ln x) \Big]_2^b$$

$$\lim_{b \to \infty} \Big(\underbrace{\ln(\ln b)}_{\infty} - \underbrace{\ln(\ln 2)}_{\#} \Big) = \infty$$

Diverges.

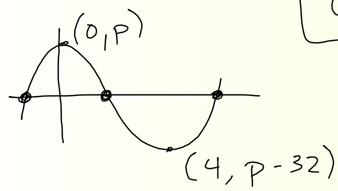
HW: p. 585

1 - 19 odd

Check FR (yellow) answers:

4. a) Max Value $= p$ when $x = 0$ Min Value $= p - 32$ when $x = 4$

b) 3 roots: $\boxed{0 < p < 32}$



c) Avg Value $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$ $\boxed{p = \dfrac{23}{4}}$

Check FR (yellow) answers:

5) a) $g(3) = \pi - \frac{1}{2}$

b) g has a relative max @ $x = 2$ where $g'(x) = f(x)$ goes from + to −

c) $y - (\pi - \frac{1}{2}) = -1(x - 3)$

d) PI's @ $x = 0$ and $x = 3$ where $g''(x) = 0$ and since $f'(x) = g''(x)$ the slopes of the graph of f change from negative $\longleftrightarrow$ positive.

Check FR (yellow) answers:

6) a) $v(t) = -16 - \frac{34}{e^{2t}}$

b) -16 ft/sec

c) $t \approx 1.070$ secs